Class 12 Mathematics

SECTION 'B'

(i) *Solution.*

$$\lim_{x\to 7}\frac{\sqrt{x}-\sqrt{7}}{x-7}=\frac{0}{0} \quad \text{form} \quad \Big\} \ \boxed{1\text{ Mark}}$$

Multiply and divide by $\sqrt{x}+\sqrt{7}$

$$\lim_{x\to 7}\frac{\sqrt{x}-\sqrt{7}}{x-7}=\lim_{x\to 7}\frac{\sqrt{x}-\sqrt{7}}{x-7}\times\frac{\sqrt{x}+\sqrt{7}}{\sqrt{x}+\sqrt{7}} \quad \Big\} \ \boxed{1\text{ Mark}}$$

$$\left.\begin{aligned}
&=\lim_{x\to 7}\frac{(\sqrt{x})^2-(\sqrt{7})^2}{(x-7)(\sqrt{x}+\sqrt{7})} \quad (\text{using } a^2-b^2=(a-b)(a+b))\\
&=\lim_{x\to 7}\frac{\cancel{x-7}}{\cancel{(x-7)}(\sqrt{x}+\sqrt{7})}\\
&=\lim_{x\to 7}\frac{1}{\sqrt{x}+\sqrt{7}}\\
&=\frac{1}{\sqrt{7}+\sqrt{7}}
\end{aligned}\right\} \ \boxed{2\text{ Marks}}$$

$$=\frac{1}{2\sqrt{7}} \quad \Big\} \ \boxed{1\text{ Mark}}$$

Hence, $\lim_{x\to 7}\frac{\sqrt{x}-\sqrt{7}}{x-7}=\frac{1}{2\sqrt{7}}$.

□

(ii) *Solution.* Given:

$$y=(3x^2-2)\cdot\sin x \tag{1}$$

$$\left.\begin{aligned}
&\text{Differentiate Eq. (1) w.r.t "}x\text{"}\\
&\frac{dy}{dx}=\frac{d}{dx}\left[(3x^2-2)\cdot\sin x\right]
\end{aligned}\right\} \ \boxed{1\text{ Mark}}$$

Using the Product Rule:

$$\frac{dy}{dx}=(3x^2-2)\cdot\frac{d}{dx}(\sin x)+\sin x\cdot\frac{d}{dx}(3x^2-2) \quad \Big\} \ \boxed{2\text{ Marks}}$$

$$\left.\begin{aligned}
&=(3x^2-2)\cdot\cos x+\sin x\cdot 3(2x)\\
&=(3x^2-2)\cdot\cos x+\sin x\cdot(6x)\\
\Rightarrow \frac{dy}{dx}&=(3x^2-2)\cos x+6x\sin x
\end{aligned}\right\} \ \boxed{2\text{ Marks}}$$

□

(iii) *Solution.* Given $f(x) = \sin x$, we compute derivatives and evaluate at $x = 0$:

$$\left.\begin{aligned}
f(x) &= \sin x && \Rightarrow f(0) = \sin(0) = 0 && \Rightarrow f(0) = 0 \\
f'(x) &= \cos x && \Rightarrow f'(0) = \cos(0) = 1 && \Rightarrow f'(0) = 1 \\
f''(x) &= -\sin x && \Rightarrow f''(0) = -\sin(0) = 0 && \Rightarrow f''(0) = 0 \\
f'''(x) &= -\cos x && \Rightarrow f'''(0) = -\cos(0) = -1 && \Rightarrow f'''(0) = -1 \\
f^{(4)}(x) &= \sin x && \Rightarrow f^{(4)}(0) = \sin(0) = 0 && \Rightarrow f^{(4)}(0) = 0 \\
f^{(5)}(x) &= \cos x && \Rightarrow f^{(5)}(0) = \cos(0) = 1 && \Rightarrow f^{(5)}(0) = 1
\end{aligned}\right\} \quad \boxed{\text{2 Marks}}$$

Using Maclaurin's Theorem:

$$\left.\begin{aligned}
f(x) &= f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(4)}(0) + \frac{x^5}{5!}f^{(5)}(0) + \cdots \\
&= 0 + x(1) + \frac{x^2}{2}(0) + \frac{x^3}{6}(-1) + \frac{x^4}{24}(0) + \frac{x^5}{120}(1) + \cdots
\end{aligned}\right\} \quad \boxed{\text{2 Marks}}$$

$$= x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots \qquad \text{is the required Maclaurin series.} \;\Big\} \quad \boxed{\text{1 Mark}}$$

□

(iv) *Solution.* Given:

$$\left.\begin{aligned}
f(t) &= \lim_{t\to 0}\left[\tan 2t\,\hat{\imath} + \ln(3+t)\,\hat{\jmath} + \hat{k}\right] \\
&= \left(\lim_{t\to 0}\tan 2t\right)\hat{\imath} + \left(\lim_{t\to 0}\ln(3+t)\right)\hat{\jmath} + \hat{k}
\end{aligned}\right\} \quad \boxed{\text{2 Marks}}$$

$$= \tan(0)\,\hat{\imath} + \ln(3+0)\,\hat{\jmath} + \hat{k} \;\Big\} \quad \boxed{\text{2 Marks}}$$

$$= 0\cdot\hat{\imath} + \ln 3\cdot\hat{\jmath} + \hat{k} \;\Big\} \quad \boxed{\text{1 Mark}}$$

□

(v) *Solution.*

$$\left.\begin{aligned} \text{Let} \quad I &= \int_1^2 \frac{x}{x^2+2}dx \\ &= \frac{1}{2}\int_1^2 \frac{2x}{x^2+2}dx \end{aligned}\right\} \quad \boxed{\text{1 Mark}}$$

$$= \frac{1}{2}\left[\ln|x^2+2|\right]_1^2 \quad \Big\} \quad \boxed{\text{2 Marks}}$$

$$\left.\begin{aligned} &= \frac{1}{2}\left[\ln\left(2^2+2\right) - \ln\left(1^2+2\right)\right] \\ &= \frac{1}{2}\left[\ln(4+2) - \ln(1+2)\right] \\ I &= \frac{1}{2}\left[\ln 6 - \ln 3\right] = \frac{1}{2}\ln\left(\tfrac{6}{3}\right) = \frac{1}{2}\ln 2 \end{aligned}\right\} \quad \boxed{\text{2 Marks}}$$

Therefore, $\displaystyle\int_1^2 \frac{x}{x^2+2}dx = \frac{1}{2}\ln 2$ □

(vi) *Solution.* Let

$$A = \int e^{2x}\cos 3x dx \tag{1}$$

Using integration by parts:

$$\left.\begin{aligned} A &= e^{2x}\int \cos 3x dx - \int\left(\frac{d}{dx}e^{2x}\cdot\int\cos 3x dx\right)dx \\ &= e^{2x}\cdot\frac{\sin 3x}{3} - \int\left(2e^{2x}\cdot\frac{\sin 3x}{3}\right)dx \\ &= \frac{e^{2x}\sin 3x}{3} - \frac{2}{3}\int e^{2x}\sin 3x dx \end{aligned}\right\} \quad \boxed{\text{1 Mark}}$$

Now apply integration by parts to the remaining integral:

$$\left.\begin{aligned} \int e^{2x}\sin 3x dx &= e^{2x}\int\sin 3x dx - \int\left(\frac{d}{dx}e^{2x}\cdot\int\sin 3x dx\right)dx \\ &= e^{2x}\left(-\frac{\cos 3x}{3}\right) - \int\left(2e^{2x}\cdot\left(-\frac{\cos 3x}{3}\right)\right)dx \\ &= -\frac{e^{2x}\cos 3x}{3} + \frac{2}{3}\int e^{2x}\cos 3x dx \\ &= -\frac{e^{2x}\cos 3x}{3} + \frac{2}{3}A \qquad \text{using (1)} \end{aligned}\right\} \quad \boxed{\text{1 Mark}}$$

Substitute back:

$$\left.\begin{aligned}
A &= \frac{e^{2x}\sin 3x}{3} - \frac{2}{3}\left(-\frac{e^{2x}\cos 3x}{3} + \frac{2}{3}A\right) \\
&= \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9} - \frac{4}{9}A \\
A + \frac{4}{9}A &= \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9} \\
\frac{13}{9}A &= \frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9} \\
\cancel{9}\cdot\frac{13}{\cancel{9}}A &= 9\left(\frac{e^{2x}\sin 3x}{3} + \frac{2e^{2x}\cos 3x}{9}\right) \quad \text{Multiply both sides by 9} \\
13A &= 3e^{2x}\sin 3x + 2e^{2x}\cos 3x \\
A &= \frac{1}{13}\left(3e^{2x}\sin 3x + 2e^{2x}\cos 3x\right) \quad \text{Divide both sides by 13} \\
A &= \frac{1}{13}\left(3e^{2x}\sin 3x + 2e^{2x}\cos 3x\right) + C
\end{aligned}\right\} \boxed{\text{2 Marks}}$$

$$\text{Therefore,} \quad \int e^{2x}\cos 3x\,dx = \frac{1}{13}\left(3e^{2x}\sin 3x + 2e^{2x}\cos 3x\right) + C \quad \Big\} \boxed{\text{1 Mark}}$$

□

(vii) *Solution.* As 2 and -4 are respectively the x and y-intercepts of the required line. Using the double-intercept form of equation of straight line:

$$\left.\frac{x}{a} + \frac{y}{b} = 1\right\} \quad \boxed{\text{2 Marks}}$$

where $a = 2$ and $b = -4$:

$$\frac{x}{2} + \frac{y}{-4} = 1 \qquad \Rightarrow \qquad \left.\frac{x}{2} - \frac{y}{4} = 1\right\} \quad \boxed{\text{2 Marks}}$$

Multiply both sides by -4:

$$\left.\begin{aligned}
&-2x + y = -4 \\
&\qquad \text{or} \\
&y = 2x - 4 \qquad \text{is the required equation of straight line.}
\end{aligned}\right\} \boxed{\text{1 Mark}}$$

□

(viii) *Solution.* Given:

$$\begin{aligned}
A(x_1, y_1) &= (1, 4) \\
B(x_2, y_2) &= (2, -3) \\
C(x_3, y_3) &= (3, 10)
\end{aligned}$$

Using the determinant formula for area:

$$\text{Area of Triangular region} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \Bigg\} \quad \boxed{\text{2 Marks}}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 2 & -3 & 1 \\ 3 & 10 & 1 \end{vmatrix} \qquad \text{(Expanding along the first row)}$$

$$= \frac{1}{2}\left[1 \cdot \begin{vmatrix} -3 & 1 \\ 10 & 1 \end{vmatrix} - 4 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & -3 \\ 3 & 10 \end{vmatrix}\right] \quad \Big\} \quad \boxed{\text{2 Marks}}$$

$$= \frac{1}{2}\Big[1 \cdot (-3 \cdot 1 - 1 \cdot 10) - 4 \cdot (2 \cdot 1 - 1 \cdot 3) + 1 \cdot (2 \cdot 10 - (-3) \cdot 3)\Big]$$

$$= \frac{1}{2}\Big[(-3 - 10) - 4(2 - 3) + (20 + 9)\Big]$$

$$= \frac{1}{2}\Big[-13 + 4 + 29\Big] = \frac{1}{2} \cdot 20 = 10 \quad \Big\} \quad \boxed{\text{1 Mark}}$$

The area of the triangle is 10 square units. □

(ix) *Solution.*

$$\left.\begin{array}{r} \text{Given equation of circle is:} \\ x^2 + y^2 + 12x - 10y = 0 \end{array}\right\} \quad \boxed{\text{1 Mark}} \qquad (1)$$

Req: center= ?, Radius = ?

Comparing Eq. (1) with

$$\left.\begin{array}{r} x^2 + y^2 + 2gx + 2fy + c = 0 \\ \Rightarrow 2g = 12 \quad \Rightarrow \quad g = 6 \\ 2f = -10 \quad \Rightarrow \quad f = -5 \\ c = 0 \end{array}\right\} \quad \boxed{\text{2 Marks}}$$

$$\left.\begin{array}{l} \text{Center} = (-g, -f) = (-6, 5) \\ \text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{(6)^2 + (-5)^2 - 0} = \sqrt{61} \end{array}\right\} \quad \boxed{\text{2 Marks}}$$

□

(x) *Solution.*

$$\left.\begin{array}{l} \text{Given:} \quad \text{Focus } F(-3, 4), \quad e = 1, \\ \text{Directrix: } 3x - 4y + 5 = 0 \end{array}\right\} \quad \boxed{\text{1 Mark}}$$

Let $P(x, y)$ be any point on the parabola. then by definition

$$\left.\begin{aligned}&\frac{|PF|}{|PM|} = e\\&\Rightarrow \frac{|PF|}{|PM|} = 1 \quad \Rightarrow \quad |PF| = |PM|\\&\text{Distance to focus: } |PF| = \sqrt{(x+3)^2 + (y-4)^2}\\&\text{Distance to directrix: } |PM| = \frac{|3x-4y+5|}{\sqrt{3^2+(-4)^2}} = \frac{|3x-4y+5|}{5}\end{aligned}\right\} \boxed{\text{2 Marks}}$$

$$\left.\begin{aligned}&\Rightarrow \sqrt{(x+3)^2 + (y-4)^2} = \frac{|3x-4y+5|}{5}\\&\text{Square both sides: } (x+3)^2 + (y-4)^2 = \frac{(3x-4y+5)^2}{25}\\&25[(x+3)^2 + (y-4)^2] = (3x-4y+5)^2 \quad \text{Multiply both sides by 25}\\&\text{Expand: } \; 25[x^2+6x+9+y^2-8y+16] = 9x^2+16y^2+25-24xy+30x-40y\\&25x^2+150x+225+25y^2-200y+400 = 9x^2+16y^2+25-24xy+30x-40y\\&\text{Bring all terms to one side:}\\&25x^2-9x^2+25y^2-16y^2+150x-30x-200y+40y+225+400-25+24xy = 0\\&16x^2+9y^2+120x-160y+600+24xy = 0\end{aligned}\right\} \boxed{\text{2 Marks}}$$

This is the equation of the required parabola. □

(xi) *Solution.*

$$\left.\begin{aligned}\text{Given:} \quad \text{foci} &= (\pm 2, 0)\\ \Rightarrow c = 2, \quad e &= \frac{1}{2}\end{aligned}\right\} \boxed{\text{1 Mark}}$$

Required: Equation of ellipse = ?, and Equation of directrices = ?

We know that

$$\left.\begin{aligned}a = \frac{c}{e} = \frac{2}{1/2} = 4 \quad &\Rightarrow a^2 = 16\\ b^2 = a^2 - c^2 = 16 - 4 = 12 &\end{aligned}\right\} \boxed{\text{2 Marks}}$$

The required equation of ellipse as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

Putting the values of a^2 and b^2 in Eq. (1),

$$\left.\frac{x^2}{16} + \frac{y^2}{12} = 1 \quad\right\} \boxed{\text{1 Mark}}$$

The directrices of the ellipse are given by $x = \pm\frac{a}{e}$:

$$\left.x = \pm\frac{4}{1/2} = \pm 8\right\} \boxed{\text{1 Mark}}$$

or $x \pm 8 = 0$ are the required equation of directrices of the ellipse. □

(xii) *Solution.* Given: $\frac{dy}{dx} + \cos 2x + 1 = 0$

$$\left.\begin{aligned} \frac{dy}{dx} &= -\cos 2x - 1 \\ dy &= (-\cos 2x - 1)dx \end{aligned}\right\} \boxed{\text{2 Marks}}$$

$$\left.\begin{aligned} \int dy &= \int (-\cos 2x - 1)dx \\ y &= -\int \cos 2x\, dx - \int dx \end{aligned}\right\} \boxed{\text{2 Marks}}$$

$$\left. y = -\frac{\sin 2x}{2} - x + C \right\} \boxed{\text{1 Marks}}$$

This is the general solution to the differential equation. □

(xiii) *Solution.*

$$\text{Given:} \quad \left.\begin{aligned} f(x) &= \sin x, \quad x_0 = -2 \\ f'(x) &= \cos x \end{aligned}\right\} \boxed{\text{1 Mark}}$$

The Newton-Raphson iteration formula is given by:

$$\left. x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\sin x_n}{\cos x_n} = x_n - \tan x_n \right\} \boxed{\text{1 Mark}}$$

We perform four iterations:

Iteration 1: $x_0 = -2$

$f(x_0) = \sin(-2) \approx -0.90930, \quad f'(x_0) = \cos(-2) \approx -0.41615,$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -4.18504$

Iteration 2: $x_1 = -4.18504$

$f(x_1) = \sin(-4.18504) \approx 0.86415, \quad f'(x_1) = \cos(-4.18504) \approx -0.50324,$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -2.46792$

Iteration 3: $x_2 = -2.46792$

$f(x_2) = \sin(-2.46792) \approx -0.62388, \quad f'(x_2) = \cos(-2.46792) \approx -0.78154,$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -3.26622$

Iteration 4: $x_3 = -3.26622$

$f(x_3) = \sin(-3.26622) \approx 0.12430, \quad f'(x_3) = \cos(-3.26622) \approx -0.99224,$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -3.14095$

$\boxed{\text{2 Marks}}$

After four iterations, the approximate root is $x_4 \approx -3.14095$ } 1 Mark

□

SECTION 'C'

Question 3

(i) *Solution.*

$$\left.\begin{aligned} \text{Given: } f(x) &= \sqrt{x+4} \\ \text{To find: } f(x^2+4) \end{aligned}\right\} \boxed{1 \text{ Mark}}$$

$$f(x^2+4) = \sqrt{(x^2+4)+4} \quad \Big\} \boxed{2 \text{ Marks}}$$

$$\left.\begin{aligned} &= \sqrt{x^2+8} \\ \text{Therefore,} \quad f(x^2+4) &= \sqrt{x^2+8} \end{aligned}\right\} \boxed{2 \text{ Marks}}$$

□

(ii) *Solution.* Given: $f(x) = x^2 + 2$

To find $f'(x)$ by definition.

By definition of derivative:

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \quad \Big\} \boxed{1 \text{ Mark}}$$

$$\left.\begin{aligned} f'(x) &= \lim_{\delta x \to 0} \frac{(x+\delta x)^2 + 2 - (x^2+2)}{\delta x} \\ &= \lim_{\delta x \to 0} \frac{\cancel{x^2} + (\delta x)^2 + 2x\delta x + \cancel{2} - \cancel{x^2} - \cancel{2}}{\delta x} \end{aligned}\right\} \boxed{2 \text{ Marks}}$$

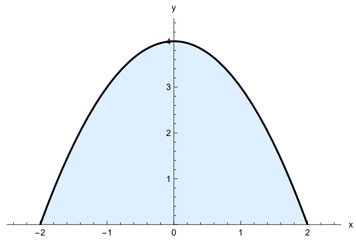
$$\left.\begin{aligned} f'(x) &= \lim_{\delta x \to 0} \frac{(\delta x)^2 + 2x\delta x}{\delta x} \\ &= \lim_{\delta x \to 0} \frac{\cancel{\delta x}(\delta x + 2x)}{\cancel{\delta x}} \\ &= \lim_{\delta x \to 0} (\delta x + 2x) \\ &= 0 + 2x \\ &= 2x \end{aligned}\right\} \boxed{2 \text{ Marks}}$$

Therefore, the derivative is $f'(x) = 2x$. □

Question 4

(i) *Solution.* Given curve as $y = 4 - x^2$

First, find the points where the curve intersects the x-axis by setting $y = 0$, we have:



$$4 - x^2 = 0 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$$

So the curve intersects the x-axis at $(-2, 0)$ and $(2, 0)$.

(1 Mark)

The required area is given by:

$$\left.\begin{aligned} \text{Area} &= \int_{-2}^{2} (4 - x^2)dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} \\ &= \left(4(2) - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right) \end{aligned}\right\} \boxed{\text{2 Marks}}$$

$$\left.\begin{aligned} &= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) \\ &= \left(\frac{24 - 8}{3}\right) - \left(\frac{-24 + 8}{3}\right) \\ &= \left(\frac{16}{3}\right) - \left(-\frac{16}{3}\right) \\ &= \frac{16}{3} + \frac{16}{3} = \frac{32}{3} \end{aligned}\right\} \boxed{\text{2 Marks}}$$

rea under the curve is $\frac{32}{3}$ square units. □

(ii) *Solution.*

$$\left.\begin{aligned} \text{Given: Slope of the line, } m_1 &= -6 \\ \text{Slope of the required line, } m_2 &= -\frac{1}{m_1} = \frac{1}{6} \\ \text{y-intercept, } c &= \frac{4}{3} \end{aligned}\right\} \boxed{\text{2 Marks}}$$

The required equation of the line is:

$$\left.\begin{aligned} y &= m_2 x + c \\ y &= \frac{1}{6}x + \frac{4}{3} \end{aligned}\right\} \boxed{\text{2 Marks}}$$

Multiply both sides by 6 to eliminate denominators:

$$6y = x + 8 \quad \Big\} \quad \boxed{\text{1 Mark}}$$

Therefore, the equation of the line is $x - 6y + 8 = 0$. □

Question 5

(i) *Solution.*

$$\left.\begin{aligned} \text{Given: } (x_1, y_1) &= \left(-\sqrt{13}, \frac{9}{2}\right) \\ \text{Hyperbola: } \frac{x^2}{4} - \frac{y^2}{9} &= 1 \\ \Rightarrow a^2 = 4, \quad b^2 &= 9 \end{aligned}\right\} \boxed{\text{1 Mark}}$$

Req.

$$\text{Equation of Tangent} = ?$$

$$\text{Equation of Normal} = ?$$

Equation of Tangent

$$\left.\begin{aligned} &\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \\ &\text{Substituting the values:} \\ &\frac{x(-\sqrt{13})}{4} - \frac{y(9/2)}{9} = 1 \\ &\Rightarrow -\frac{\sqrt{13}x}{4} - \frac{y}{2} = 1 \\ &\text{Multiplying both sides by } -1: \\ &\frac{\sqrt{13}x}{4} + \frac{y}{2} = -1 \\ &\Rightarrow \frac{y}{2} = -\frac{\sqrt{13}x}{4} - 1 \\ &\Rightarrow y = -\frac{\sqrt{13}x}{2} - 2 \quad \text{is the Req Equation of Tangent line.} \end{aligned}\right\} \boxed{\text{2 Marks}}$$

Equation of Normal

$$\left.\begin{aligned} &y - y_1 = -\frac{a^2 y_1}{b^2 x_1}(x - x_1) \\ &\text{Substituting the values:} \\ &y - \frac{9}{2} = \frac{\cancel{-18}\,2}{\cancel{-9}\sqrt{13}}(x + \sqrt{13}) \\ &y - \frac{9}{2} = \frac{2}{\sqrt{13}}(x + \sqrt{13}) \\ &y - \frac{9}{2} = \frac{2}{\sqrt{13}}x + \frac{2\cancel{\sqrt{13}}}{\cancel{\sqrt{13}}} \\ &y - \frac{9}{2} = \frac{2}{\sqrt{13}}x + 2 \\ &\Rightarrow y = \frac{2}{\sqrt{13}}x + 2 + \frac{9}{2} \\ &\Rightarrow y = \frac{2}{\sqrt{13}}x + \frac{13}{2} \quad \text{is the Required Equation of Normal.} \end{aligned}\right\} \boxed{\text{2 Marks}}$$

□

(ii) *Solution.*

$$\left.\begin{aligned} \text{Center of circle: } (h,k) &= (0,0) \\ \text{Point lies on circle: } (1,2) & \end{aligned}\right\} \boxed{\text{1 Mark}}$$

We know that the equations of Circle as;

$$\left.\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 \\ \Rightarrow (x-0)^2+(y-0)^2 &= r^2 \\ \Rightarrow x^2+y^2 &= r^2 \end{aligned}\right\} \boxed{\text{2 Marks}} \tag{1}$$

since $(1,2)$ lies on circle

$$\left.\begin{aligned} &\therefore (1) \Rightarrow 1^2+2^2 = r^2 \Rightarrow r^2 = 5 \quad \text{(putting in (1))} \\ &\therefore (1) \Rightarrow 1^2+2^2 = 5 \end{aligned}\right\} \boxed{\text{2 Marks}}$$

□

is the required equation of the circle whose center is $(0,0)$ and contains the point $(1,2)$

Question 6

(i) *Solution.*

$$\left.\begin{aligned} \text{Given: } f(x,y) &= x^2y^3\tan^{-1}y \\ \text{Required: } f_x &=? \quad \text{and} \quad f_y =? \end{aligned}\right\} \boxed{\text{1 Mark}}$$

Partial derivative with respect to x:

$$\left.\begin{aligned} f_x &= \frac{\partial}{\partial x}(x^2y^3\tan^{-1}y) \\ &= y^3\tan^{-1}y\cdot\frac{\partial}{\partial x}(x^2) \\ &= y^3\tan^{-1}y\cdot 2x \\ &\text{or} \\ f_x &= 2xy^3\tan^{-1}y \end{aligned}\right\} \boxed{\text{2 Marks}}$$

Partial derivative with respect to y:

$$\left.\begin{aligned} f_y &= \frac{\partial f(x,y)}{\partial y} = \frac{\partial}{\partial y}\left(x^2y^3\tan^{-1}y\right) \\ &= x^2\frac{\partial}{\partial y}\left(y^3\tan^{-1}y\right) \qquad \text{(treating } x \text{ as constant)} \\ &= x^2\left[y^3\cdot\frac{\partial tan^{-1}y}{\partial y}+\tan^{-1}y\cdot\frac{\partial(y^3)}{\partial y}\right] \qquad \text{(product rule)} \\ &= x^2\left[y^3\cdot\frac{1}{1+y^2}+\tan^{-1}y\cdot 3y^2\right] \qquad \because \frac{\partial tan^{-1}y}{\partial y} = \frac{1}{1+y^2} \\ f_y &= x^2y^2\left[\frac{y}{1+y^2}+3\tan^{-1}y\right] \end{aligned}\right\} \boxed{\text{2 Marks}}$$

□

(ii) *Solution.*

$$\left.\begin{aligned} \text{Given interval: } [a,b] &= [1,3] \\ \text{Width for } n = 6 \text{ subintervals: } \Delta x &= \frac{b-a}{n} = \frac{3-1}{6} = \frac{2}{6} = 0.3333 \end{aligned}\right\} \boxed{\text{1 Mark}}$$

$$\left.\begin{aligned} &\text{Trapezoidal rule for } \Delta x = 0.3333 \text{ and } n = 6: \\ &\qquad \int_1^3 x^2\,dx \approx T_6 = \frac{h}{2}\left[f_0 + 2(f_1 + f_2 + f_3 + f_4 + f_5) + f_6\right] \end{aligned}\right\} \boxed{\text{1 Mark}}$$

The function values for $f_0, f_1, f_2, f_3, f_4, f_5, f_6$ at grid points $x_0, x_1, x_2, x_3, x_4, x_5, x_6$ are

$$\left.\begin{aligned} &x_0 = a = 1 && \Rightarrow f(x_0) = 1^2 = 1 \Rightarrow f_0 = 1 \\ &x_1 = a + \Delta x = 1 + 0.3333 = 1.3333 && \Rightarrow f(x_1) = (1.3333)^2 = 1.7689 \Rightarrow f_1 = 1.7689 \\ &x_2 = a + 2\Delta x = 1 + 2(0.3333) = 1.6666 && \Rightarrow f(x_2) = (1.6666)^2 = 2.7766 \Rightarrow f_2 = 2.7766 \\ &x_3 = a + 3\Delta x = 1 + 3(0.3333) = 2 && \Rightarrow f(x_3) = 2^2 = 4 \Rightarrow f_3 = 4 \\ &x_4 = a + 4\Delta x = 1 + 4(0.3333) = 2.3333 && \Rightarrow f(x_4) = (2.3333)^2 = 5.4443 \Rightarrow f_4 = 5.4443 \\ &x_5 = a + 5\Delta x = 1 + 5(0.3333) = 2.6666 && \Rightarrow f(x_5) = (2.6666)^2 = 7.1108 \Rightarrow f_5 = 7.1108 \\ &x_6 = a + 6\Delta x = 1 + 6(0.3333) = 3 && \Rightarrow f(x_6) = 3^2 = 9 \Rightarrow f_6 = 9 \end{aligned}\right\} \boxed{\text{2 Marks}}$$

Now substitute into the formula:

$$\left.\begin{aligned} T_6 &= \frac{0.3333}{2}\left[1 + 2(1.7689 + 2.7766 + 4 + 5.4443 + 7.1108) + 9\right] \\ &= \frac{0.3333}{2}\left[1 + 42.2012 + 9\right] \\ T_6 &= 8.699 \end{aligned}\right\} \boxed{\text{1 Mark}}$$

Therefore, the approximate value of the integral is 8.699. □